Nonperturbative Gluon Radiation and Energy Dependence of Elastic Scattering

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The energy dependence of the total hadronic cross sections is caused by gluon bremsstrahlung which we treat nonperturbatively. It is located at small transverse distances about $0.3\,fm$ from the valence quarks. The cross section of gluon radiation is predicted to exponentiate and rise with energy as s^{Δ} with $\Delta = 0.17 \pm 0.01$. The total cross section also includes a large energy independent Born term which corresponds to no gluon radiation. The calculated total cross section and the slope of elastic scattering are in good agreement with the data.

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The dynamics of energy dependence of the hadronic total cross sections is a long standing challenge since 1973 when this effect was first observed at the ISR. In DIS the source for the rising total cross section for interaction of highly virtual photons is well understood in QCD as caused by an intensive gluon bremsstrahlung [1,2]. Indeed, radiation of each gluon supply an extra $\ln s$ and $\ln Q^2$. This is a specific regime of radiation when a $\bar{q}q$ fluctuation of the photon of a tiny size $\sim 1/Q$ radiates gluons at much larger transverse separations.

It is difficult to extend the perturbative results to soft hadronic collisions because it is quite a different regime where the approximations made in the perturbative case break down. Namely, gluon radiation giving rise to the energy dependence of the total cross section occurs at rather small transverse distances around the valence quarks, $r_0 \approx 0.3\,fm$ which are much smaller than the mean interquark spacing in light hadrons. This conclusion follows from the analysis [3] of the data for diffractive gluon radiation based on the light-cone approach when the effective nonperturbative interaction of radiated gluons is included.

The smallness of the gluon clouds of the valence quarks is confirmed by the study of the gluon formfactor of the proton employing QCD sum rules [4]. The Q^2 dependence of the formfactor turns out to be rather weak corresponding to a small radius of the gluon distribution which was estimated at the same value $r_0 \approx 0.3\,fm$.

Another evidence for a short gluon-gluon correlation length $\lambda \approx 0.3\,fm$ arises in the stochastic vacuum model of Dosch and Simonov [5,6], as it was measured on the lattice [7]. In the case of a poorly populated gluon cloud (only about one gluon is radiated by a valence quark at available energies, see below) this corresponds to the correlation radius between the quark and the gluon.

The same size $\sim 0.3\,fm$ emerges from the Shuryak's instanton liquid model [8] as the instanton size which controls the mean radius of the sea surrounding a valence quark and by many phenomenological analyses.

In the Gribov's theory of confinement [9,10] the same distance $\sim 0.3\,fm$ should correspond to the critical regime related to breaking of chiral symmetry. Namely, at smaller distances, a perturbative quark-gluon basis is appropriate, while at larger separations quasi-Goldstone pions emerge. The corresponding critical value of the QCD running constant $\alpha_c = 0.43$ evaluated in [9] turns out to be very close to our estimate (see below) of α_s corresponding to gluon radiation separated by $0.3\,fm$. The value of α_s is crucial for our evaluation of the energy dependence of the gluon bremsstrahlung.

It is quite plausible that all these observations are the manifestations of the same dynamics, however it is still unclear how to make a Lorentz boost in these approaches. This is the advantage of the light cone treatment of nonperturbative gluon radiation [3] which seems to be best designed for calculating the energy dependence of the total cross section. We believe that the nonperturbative interaction of gluons introduced in [3] as a light-cone potential is an effective manifestation of properties of the QCD vacuum. Similar scale $\sim 0.3\,fm$ found in all these approaches supports this conjecture.

An interesting attempt to implement the nonperturbative gluon interaction into the Pomeron ladder building was made recently by Kharzeev and Levin [14] and Shuryak [15]. They found that the radiation of colorless pairs of gluons is a part of the leading-log approximation since each extra power of the coupling α_s cancels due to the strong glue-glue interaction. The radiated glueballs are not clustering around the valence quarks, but spreading all over the hadron. The estimated $\Delta \approx 0.05$ [15] is about twice as small (and even more so if corrected for unitarity) as the data need. Although the scale for $\alpha'_P \sim 1/M_0^2$ seems to be correct, an extra factor $\Delta/4$ makes it too small.

We start calculating the energy dependence of the total cross section summing up the contributions of different Fock components of the incident hadron,

$$\sigma_{tot}^{hN} = \sum_{n=0} \sigma_n^{hN} \; . \tag{1}$$

To avoid double-counting, we sum over cross sections σ_n of physical processes corresponding to the radiation of n gluons.

The lowest Fock component of a hadron contains only valence quarks. The corresponding Born term in the total cross section has the form (for the sake of simplicity we assume that the incident hadron is a meson),

$$\sigma_0^{hN} = \int_0^1 d\alpha_q \int d^2R \left|\Psi^h_{\bar{q}q}(\alpha_q, R)\right|^2 \sigma^N_{\bar{q}q}(R) \ . \qquad (2)$$

Here the Fock state wave function $\Psi^h_{\bar{q}q}(\alpha_q, R)$ depends on the transverse $q-\bar{q}$ separation R and on the fraction α_q of the light-cone momentum of the pair carried by the quark. The cross section $\sigma^N_{\bar{q}q}(R)$ of interaction of the valence $\bar{q}q$ dipole with a nucleon cannot be calculated perturbatively since the separation R is large. According to [13] this energy independent term has no relation to the smallness of the spots (gluon clouds) in the hadron.

The next contribution to σ^{hN}_{tot} comes from the radiation of a single gluon. The radiation is possible only due to the difference between the cross sections for the $\bar{q}q$ and $\bar{q}qG$ Fock components, otherwise no new state can be produced [3]. The cross section of radiation of a single gluon reads [3],

$$\sigma_1^{hN} = \int_0^1 d\alpha_q \int d^2R \left|\Psi^h_{\bar{q}q}(R, \alpha_q)\right|^2$$
$$\times \frac{9}{4} \int\limits_{\alpha_G \ll 1} \frac{d\alpha_G}{\alpha_G} \int d^2r \Bigg\{ \left|\Psi_{\bar{q}G}(\vec{R}+\vec{r}, \alpha_G)\right|^2 \sigma^N_{\bar{q}q}(\vec{R}+\vec{r})$$
$$+ \left|\Psi_{qG}(\vec{r}, \alpha_G)\right|^2 \sigma^N_{\bar{q}q}(r) - \mathrm{Re}\,\Psi^*_{qG}(\vec{r}, \alpha_G)\,\Psi_{\bar{q}G}(\vec{R}+\vec{r}, \alpha_G)$$
$$\times \left[\sigma^N_{\bar{q}q}(\vec{R}+\vec{r}) + \sigma^N_{\bar{q}q}(r) - \sigma^N_{\bar{q}q}(R)\right] \Bigg\} \qquad (3)$$

Here α_G is the fraction of the quark momentum carried by the gluon, and $\vec{r}$ is the quark-gluon transverse separation. The three terms in the curly brackets correspond to the radiation of the gluon by the quark, by the antiquark and to their interference respectively.

The nonperturbative wave function for a quark-gluon Fock component is derived in [3]. Neglecting the quark mass, the wave function reads,

$$\Psi_{qG}(\vec{r}, \alpha_G \ll 1) = -\frac{2\,i}{\pi} \sqrt{\frac{\alpha_s}{3}}\ \frac{\vec{e}^{\,*} \cdot \vec{r}}{r^2}\, e^{-r^2 b_0^2/2} \ , \qquad (4)$$

where $\vec{e}$ is the polarization vector of the massless gluon. The parameter $b_0 = 0.65\,GeV$ characterizing the nonperturbative quark gluon interaction is fixed by the data on large mass diffractive dissociation corresponding to the triple-Pomeron limit. It leads to quite a short mean quark-gluon separation $r_0 = \sqrt{\langle r^2 \rangle} = 1/b_0 \approx 0.3\,fm$, which is small relative to the hadronic size. Therefore, only one or the other of the first two terms in (3) can be large, while the interference one can always be neglected. In this case, the integration in (3) is easily performed,

$$\sigma_1^{hN} = N\,\frac{4\,\alpha_s}{3\,\pi}\,\ln\left(\frac{s}{s_0}\right)\ \frac{9\,C}{4\,b_0^2} \ . \qquad (5)$$

Here we assume that the approximation $\sigma^N_{\bar{q}q}(r) = Cr^2$ is valid for $r \sim 1/b_0$. N is the number of valence quarks, $\ln(s/s_0) = \ln[(\alpha_G)_{max}/(\alpha_G)_{min}]$, where $(\alpha_G)_{min} = 2\,b_0^2/s$, but $(\alpha_G)_{max}$ is ill defined. It should be sufficiently small to use the wave functions (3). This leads to the condition to $s_0 \gg 3\,GeV^2$. At high energy σ_1 has little sensitivity on s_0 which we fix at $s_0 = 30\,GeV^2$ for further applications.

The radiation of each new, n-th gluon can be treated as radiation by a color triplet which is an effective quark surrounded by $n-1$ gluons. It should be resolved by the soft interaction with the target to be different from the radiation of $n-1$ gluons *i.e.* the radiation cross section is proportional to the difference between the total cross sections of the two subsequent Fock states which is $9C/4b_0^2$. This can be also proved using a $1/N_c$ expansion and the dipole representation of Mueller [16]. Since the radiation of a gluon with $\alpha_G \ll 1$ does not affect the impact parameter of the radiating quark, all the quark lines in the final state cancel with the same lines in the initial state (see the prescription for calculating the radiative cross section in [17]), except for the radiation of the n-th gluon. Thus, σ_n for quark-proton interaction in the leading-log approximation reads,

$$\sigma_n^{qN} = \frac{1}{n!}\,\left[\frac{4\,\alpha_s}{3\,\pi}\,\ln\left(\frac{s}{s_0}\right)\right]^n\ \frac{9\,C}{4\,b_0^2} \ . \qquad (6)$$

Summing up the powers of logarithms in (1) we arrive at the following expression for the total cross section,

$$\sigma^{hp}_{tot} = \tilde{\sigma}^{hp}_0 + N\,\frac{9\,C}{4\,b_0^2}\ \left(\frac{s}{s_0}\right)^{\Delta} \ , \qquad (7)$$

with

$$\Delta = \frac{4\,\alpha_s}{3\,\pi} \ , \qquad (8)$$

and $\tilde{\sigma}^{hp}_0 = \sigma^{hp}_0 - 9C/4b_0^2$. The soft Pomeron intercept, $\alpha_P(0) = 1 + \Delta$, and can be evaluated provided that the QCD coupling α_s is known.

In Gribov's confinement scenario, chiral symmetry breaking occurs when the running coupling α_s exceeds the critical value $\alpha_s = \alpha_c \approx 0.43$ [9]. This should happen at a distance of the order of the size of a constituent quark $\sim 0.3\,fm$. Therefore, this value can be used in (8).

One can also calculate the mean $\langle\alpha_s\rangle$ for nonperturvative gluon radiation averaging over transverse momenta k_T of the radiated gluons. The popular way to extend the running QCD coupling $\alpha_s(k_T^2)$ down to small k_T is a

shift of the variable $k_T^2 \Rightarrow k_T^2 + k_0^2$, where $k_0^2 \approx 0.25\,GeV^2$ was evaluated in [10] using the dispersive approach to calculating higher twist effects in hard reactions [18]. The nonperturbative interaction of the radiated gluons drastically suppresses small transverse momenta, pushing $\langle k_T^2\rangle$ to higher values which lowers α_s. We use the transverse momentum gluon distribution calculated in [3] in the light-cone approach in terms of the universal color dipole cross section [11]. We calculated $\langle\alpha_s\rangle$ with a simple parameterization $\sigma(\rho) \propto 1 - \exp(\rho^2/\rho_0^2)$. For a reasonable variation of $\rho_0 = 0.3 - 1\,fm$ the mean coupling is in the range $\langle\alpha_s\rangle = 0.38 - 0.43$ which is very close the the critical value mentioned above [9]. Taking the mid value $\langle\alpha_s\rangle = 0.4$ we get from (8),

$$\Delta = 0.17 \pm 0.01\ . \tag{9}$$

This value is about twice as large as the one suggested by the data for the energy dependence of total hadronic cross sections [12]. However, the radiative part is a rather small fraction of the total cross section (at medium high energies). A structure similar to (7) with a large Δ was suggested in [24] (with quite a different motivation) and proved to agree well with the data.

The factor C in the second term in (7) can also be evaluated. We calculated the dipole cross section with the gluon effective mass $0.15\,GeV$ (to incorporate confinement) and $\alpha_s = 0.4$ and found $C = 2.3$ at $\rho = 1/b_0$. Thus, the energy dependent term in (7) is fully determined.

The cross section (7) apparently violates the Froissart bound and one should perform unitarity corrections. Indeed, the partial elastic amplitude shows a precocious onset of unitarity restrictions at small impact parameters important even at medium high energies [19].

Following [12,20] we assume that the t-dependence of the pp elastic amplitude is given by the Dirac electromagnetic formfactor squared. Correspondingly, the mean square radius $\langle\tilde r_{ch}^2\rangle$ evaluated in [20] should be smaller than $\langle r_{ch}^2\rangle$.

For the dipole parameterization of the formfactor the partial elastic amplitude which is related via unitarity to σ_n^{pp}, given by (2), (6), takes the form,

$$\mathrm{Im}\,\gamma_n^{pp}(b,s) = \frac{\sigma_n^{pp}(s)}{8\,\pi\,B_n}\,y^3\,K_3(y)\ , \tag{10}$$

where $K_3(y)$ is the third order modified Bessel function and $y = b\sqrt{8/B_n}$. The slope parameter grows linearly with n due to the random walk of radiated gluons with a step $1/b_0^2$ in the impact parameter plane, $B_n = 2\langle\tilde r_{ch}^2\rangle/3 + n/2b_0^2$.

We unitarize the partial amplitude $\mathrm{Im}\,\gamma_P(s,b) = \sum\limits_{n=0} \mathrm{Im}\,\gamma_n(s,b)$ using the quasi-eikonal model [21],

$$\mathrm{Im}\,\Gamma_P(b,s) = \frac{1 - \exp\left[-D(s)\,\mathrm{Im}\,\gamma_P(b,s)\right]}{D(s)} \tag{11}$$

where $D(s) - 1 = \sigma_{sd}(s)/\sigma_{el}(s)$ is the ratio of the single diffractive to elastic cross sections. It is approximately equal to 0.25 at the lowest ISR energy and slightly decreases with energy $\propto s^{-0.04}$ [22,23]. Note that good results can be also achieved with a different unitarization scheme similar to one suggested in [24]. The details will be presented elsewhere.

In order to calculate the total cross section, $\sigma_{tot} = 2\int d^2b\,\mathrm{Im}\Gamma(b,s)$, one needs to fix the energy independent term with $n = 0$ in (10). This can be done comparing with the data for σ_{tot} at any energy sufficiently high to neglect Reggeon contributions. We used the most precise data [25] at $\sqrt{s} = 546\,GeV$ and fixed $\tilde\sigma_0 = 39.7\,mb$.

The predicted energy dependence of σ_{tot}^{pp} is shown by the dashed curve in Fig. 1 which is in good agreement with the data at high energy [26], but apparently needs Reggeon corrections towards low energies. We added a

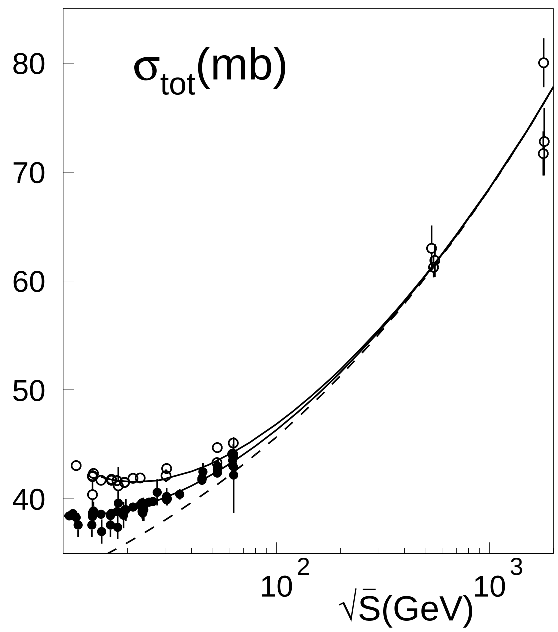

FIG. 1. Data for total pp (full circles) and $\bar pp$ (open circles) cross section [26] and the prediction of Eq. (10) for the energy dependence of the Pomeron part (dashed curve). The solid curves include Reggeon contributions fitted to the data.

Reggeon term $\mathrm{Im}\,\Gamma_R(s,b)[1 - \mathrm{Im}\,\Gamma_P(s,b)]$ screened by unitarity corrections, which was fitted independently for pp and $\bar pp$, $\sigma_R^{pp} = 17.8\,mb/\sqrt{s/s_0}$, $\sigma_R^{\bar pp} = 32.8\,mb/\sqrt{s/s_0}$. The fitted Reggeon slope is $B_R = R_R^2 + 2\,\alpha'_R\ln(s/s_0)$, where $\alpha'_R = 0.9\,GeV^{-2}$ and $R_R^2 = 3\,GeV^{-2}$.

The results are shown by the solid curves of Fig. 1 (pp bottom curve and $\bar pp$ upper curve).

As soon as the partial amplitude (11) is known, we are in position to predict the slope of elastic scattering at $t = 0$, $B_{el}(s) = \langle b^2\rangle/2$, where averaging is weighted by the partial amplitude (11). The results exhibit good agreement when compared with the pp and $\bar pp$ data [26] in Fig. 2. Although the value of the slope essentially de-

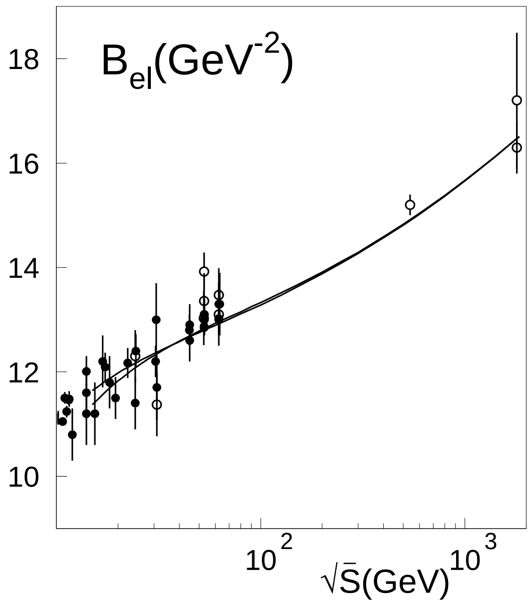

FIG. 2. Data for the elastic slope [26] and our predictions. The upper and bottom curves and open and full circles correspond to $\bar{p}p$ and pp respectively.

pends on our choice of $\langle\tilde{r}^2_{ch}\rangle$ in (10), the predicted energy dependence, *i.e.* the effective value α'_P is fully defined by the parameter b_0 fixed in [3]. Indeed, each radiated gluon makes a "step" $\sim 1/b_0^2 = (0.3\,fm)^2$ in the impact parameter plane leading to the rising energy dependence of the elastic slope. Eventually, at very high energies the approximation of small gluon clouds breaks down but the mean number of gluons in a quark $\langle n\rangle = \Delta\ln(s/s_0)$ remains quite small even in the energy range of colliders. It is only $\langle n\rangle = 0.7 - 1$ at the ISR and reaches about two gluons at the Tevatron. Correspondingly, the mean square of the quark radius grows from $0.06\,fm^2$ to $0.18\,fm^2$ which is still rather small compared to the mean square of the charge radius of the proton.

Summarizing, the strong nonperturbative interaction of radiated gluons substantially shrinks the gluon clouds around of valence quarks. This spots are small ($\sim 0.3\,fm$) compared to the hadronic radius, but the gluon radiation grows with energy as s^Δ where $\Delta = 0.17\pm0.01$. Such a steep rise does not contradict the data since this fraction of the total cross section is rather small (it contains a factor $1/b_0^2 \approx 1\,mb$). A large energy independent fraction comes from the Born term which corresponds to scattering of the valence quark skeleton without gluon radiation. A very soft interaction which cannot resolve and excite the small spots contributes to this term. It cannot be reliably predicted and is fixed by data , while the energy dependent term is fully calculated. The results are in good agreement with the data for total pp and $\bar{p}p$ cross sections and elastic slopes.

Note that although we have some room for fine-tuning in the the parameters (C, s_0, $\langle\tilde{r}^2_{ch}\rangle$), the results are rather insensitive and the agreement with data is always pretty good. We have also tried a different unitarization scheme suggested in [24] arriving to similar results.

The details of calculations and further comparison with elastic scattering data will be published elsewhere.

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